



Algorithms and Data Structures – Conditional Course

Winter Term 2026/27

Exercise Sheet 5

Due: Wednesday, 25 November 2026, 12:00

Exercise 1: Hashing with Open Addressing (5 Points)

Let $\mathcal{T}$ be a hash table of size $m = 13$. Define hash functions $h_1, h_2, h_3 : \mathbb{N}_0 \rightarrow \{0, \dots, m-1\}$ by¹

- $h_1(x) := \bar{x} \bmod m$,
- $h_2(x) := 3x \bmod m$,
- $h_3(x) := (x + 1) \bmod m$.

Insert the keys 23, 12, 75, 945, 30, 99, 345 in this order into the initially empty table. Resolve collisions using:

- (a) Linear probing with h_1 . (2 Points)
- (b) Double hashing with h_2 and h_3 , using $(h_2(x) + i \cdot h_3(x)) \bmod m$. (3 Points)

Show the table after every insertion.

Exercise 2: An Application of Hash Tables (5 Points)

Consider the following algorithm.

Algorithm 1 algorithm	▷ Input: Array A of length n with integer entries
<pre> 1: for $i = 1$ to $n - 1$ do 2: for $j = 0$ to $i - 1$ do 3: for $k = 0$ to $n - 1$ do 4: if $A[i] - A[j] = A[k]$ then 5: return true 6: return false </pre>	

- (a) Describe what **algorithm** computes and analyze its asymptotic running time. (2 Points)
Hint: The difference $|A[i] - A[j]|$ can be arbitrarily large.
- (b) Describe an alternative algorithm $\mathcal{B}$ based on hashing that solves the same problem, i.e., $\mathcal{B}(A) = \text{algorithm}(A)$ for every input A , in time $\mathcal{O}(n^2)$. Justify your answer. (3 Points)
Hint: You may assume that inserting and finding keys in a hash table takes $\mathcal{O}(1)$ time when its load factor satisfies $\alpha = \mathcal{O}(1)$.
- (c) **Bonus:** Describe another algorithm for this problem that does not use hashing and runs in $\mathcal{O}(n^2 \log n)$ time. Justify your answer. (5 Bonus Points)

¹Here $\bar{x}$ denotes the sum of the decimal digits of x .

Exercise 3: (Non-)Universal Families of Hash Functions (10 Points)

(a) Let $\mathcal{S} = \{0, \dots, M-1\}$ and define

$$\mathcal{H}_1 := \{h : x \mapsto ax^2 \bmod m \mid a \in \mathcal{S}\}.$$

Show that $\mathcal{H}_1$ is not c -universal for any fixed constant $c \geq 1$ independent of m . (4 Points)

(b) Now let m be prime and $k = \lfloor \log_m M \rfloor$. Write keys $x \in \mathcal{S}$ in base m as $x = \sum_{i=0}^k x_i m^i$. Consider the family introduced in the lecture:

$$\mathcal{H}_2 := \left\{ h : x \mapsto \sum_{i=0}^k a_i x_i \bmod m \mid a_i \in \{0, \dots, m-1\} \right\}.$$

Show that $\mathcal{H}_2$ is 1-universal. (6 Points)

Hint: Distinct keys $x \neq y$ differ in at least one base- m digit, $x_j \neq y_j$.

Additional fact: Since m is prime, every $r \in \{1, \dots, m-1\}$ has a multiplicative inverse $e \in \{1, \dots, m-1\}$ such that $re \equiv 1 \pmod{m}$.