



Algorithms and Data Structures – Conditional Course Winter Term 2026/27

Exercise Sheet 8

Due: Wednesday, 16 December 2026, 12:00

Exercise 1: Depth-First Search

(6 Points)

For each vertex v , define two timestamps:

- $t_{v,1}$: the time at which DFS colors v gray;
- $t_{v,2}$: the time at which DFS colors v black.

Consider the directed graph $G = (V, E)$ with

$$\begin{aligned} V &= \{u_1, u_2, u_3, u_4, u_5\}, \\ E &= \{(u_1, u_2), (u_1, u_3), (u_2, u_3), (u_3, u_4), (u_4, u_1), \\ &\quad (u_5, u_1), (u_5, u_3), (u_5, u_4)\}. \end{aligned}$$

- (a) Draw G . *(2 Points)*
- (b) Label each vertex with its processing interval $[t_{v,1}, t_{v,2}]$. Whenever DFS can choose among several vertices, choose the one with the smallest index; thus the search starts at u_1 . *(2 Points)*
- (c) Classify every edge as a tree, back, forward, or cross edge. *(2 Points)*

Exercise 2: Breadth-First Search on the Actor Graph (6 Points)

The file `movies.csv` contains metadata for more than 4000 films, including up to three actors per film. Define the actor graph G : each vertex represents an actor in the file, and two actors share an edge if they appeared in at least one film together. The template `bfs.py` already includes a function that constructs the adjacency list. Implement and extend BFS to solve the following tasks.¹

- (a) BFS starts at a specified vertex and explores all reachable vertices. Extend it to report each newly discovered vertex's level relative to the start: the start has level 0, its neighbors have level 1, previously unvisited neighbors of level-1 vertices have level 2, and so on. Use your implementation to determine the distance between **Tom Hanks** and **Orlando Bloom** in the actor graph. Equivalently, what level does BFS assign Orlando Bloom when started at Tom Hanks? Include your answer in your submission. (3 Points)
- (b) Write another function that uses your BFS from (a) to count all connected components. What does it mean for two actors to belong to the same connected component? Include your answer in your submission. (3 Points)
- (c) **Bonus:** How large is the largest connected component, and how small is the smallest? Report your results. (2 Bonus Points)

Exercise 3: Acyclic Graphs (8 Points)

- (a) What is the maximum number m of edges in an undirected connected graph with n vertices and no self-loops?² Justify your answer. (2 Points)
- (b) Show that every undirected connected graph without a cycle³ has exactly $n - 1$ edges, where n is its number of vertices. (3 Points)
Hint: You can use induction on $n \geq 1$.
- (c) Given an undirected connected graph $G = (V, E)$ with $n = |V|$, describe an $O(n)$ -time algorithm that decides whether G contains a cycle. Explicitly specify the data structure in which G is given.

¹Do not include `movies.csv` in your submission.

²A self-loop is an edge of the form $\{v, v\}$.

³A cycle is a path $v_1, \dots, v_k$ together with an edge $\{v_1, v_k\}$ joining its endpoints.